

UNCLASSIFIED

AD NUMBER
AD030294
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 30 SEP 1954. Other requests shall be referred to Office of Naval Research, Arlington, VA 22203.
AUTHORITY
13 Sep 1977, ST-A per ONR ltr

THIS PAGE IS UNCLASSIFIED

Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD

30294

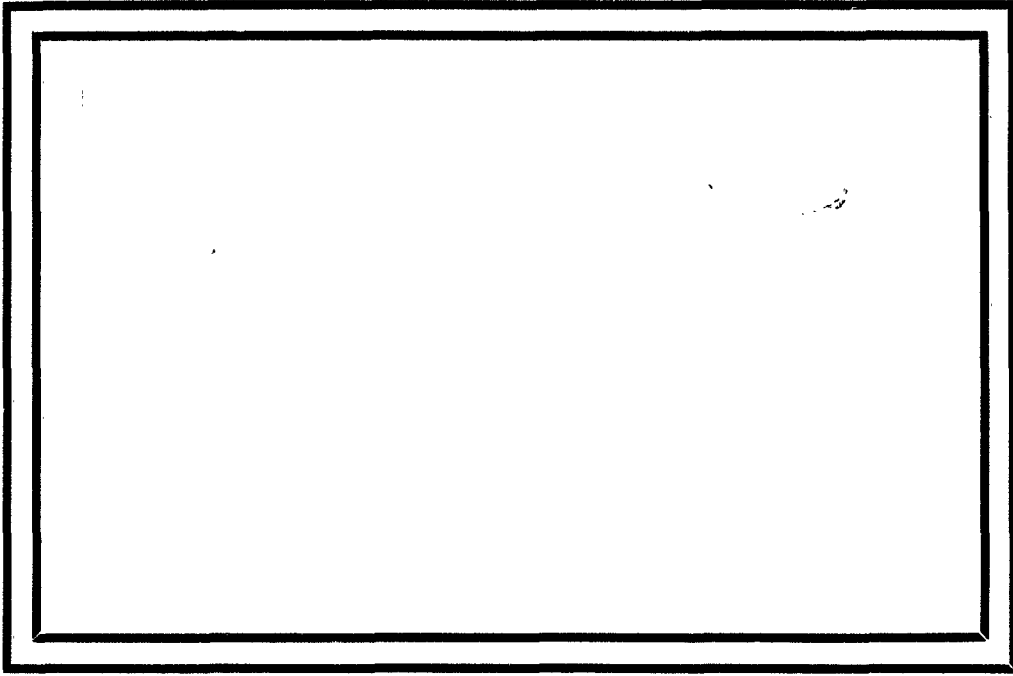
NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY ANY PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, REPRODUCE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED

AD No. 30294

ASTIA FILE COPY



University of Maryland

PHYSICS DEPARTMENT

College Park, Maryland

Reproduced

FROM LOW CONTRAST COPY.

Self-Energy Effects on Meson - Nucleon
Scattering According to the
Tamm-Dancoff Method* **

William M. Visscher

University of Maryland

College Park, Maryland

Technical Report No. 7

* Part of this paper has been submitted to the Graduate School of Cornell University in partial fulfillment of their requirements for the Ph. D. degree.

** Supported in part by the Office of Naval Research under contract Nonr 594 (00)

Abstract

The lowest order Tamm-Dancoff equations for the meson-nucleon system are derived by the method of Cini, using the wave-functions defined by Dyson. The contributions of the meson and nucleon self-energies to the kernel of the momentum-space integral equation are renormalized. Their effects are absorbed into the coupling G^2 , making it momentum and energy dependent. The effective coupling turns out to exhibit an anomalous behavior, having a pole for a space-like momentum, to which it is difficult to attach a sensible physical interpretation.

I. Introduction

Calculations of the meson-nucleon scattering phase shifts have recently been performed by the Tamm-Dancoff Method.⁽¹⁾ This paper is essentially a continuation of one in which that undertaking is reported. The lowest-order Tamm-Dancoff equations were there derived and solved approximately by numerical methods, with several of the terms in the kernel, namely, those which correspond to the nucleon, meson, and vacuum self-energies, and some which occur only in the $I = 1/2, J = 1/2$ state, simply dropped. It is the purpose of the present paper to ascertain the effects of the former omission on the scattering phase shifts; in other words to find the renormalization corrections to the scattering in states in which either I or J , or both, are different from $1/2$.

One of the omitted terms, the vacuum self-energy, may be eliminated by a simple redefinition of the Tamm-Dancoff amplitudes, as was shown by Dyson.⁽²⁾ In addition, this redefinition makes possible a consistent relativistic renormalization of mass and charge.⁽³⁾ In order to perform

-
- (1) M. Fubini *Nuovo Cimento* 10, 564; F. Dyson, S. Schweber, and W. Visscher *Phys. Rev.* 90 372.
 - (2) F. J. Dyson *Phys. Rev.* 90 994 (1953). We shall call Dyson's modification of the Tamm-Dancoff theory the DTD theory.
 - (3) F. J. Dyson *Phys. Rev.* 91 421, 1543 (1953)

a reliably unambiguous separation of the divergent parts of the self-energy integrals, it is essential to write them in a covariant form. For this purpose we use the method of Cini ⁽⁴⁾, applied to the DTD theory.

In part III we reduce the equations derived in part II to a form similar to that obtained formerly ⁽¹⁾. They turn out to be, in fact, identical, except for some of the self-energy terms, and one of the terms that occurs only in the $I = 1/2, J = 1/2$ state. Thus the self-energy corrections which will be derived in Part IV may be applied to the phase shifts calculated previously. ⁽¹⁾ The results are discussed in part V.

II. The Cini method applied to the DTD equations.

We define a two-particle amplitude

$$\phi_{\alpha}(x, y; t) = (\bar{\Psi}_0^*(t) \psi_e(x) \phi_{\alpha}(y) \bar{\Psi}(t)) \quad (1)$$

where $\bar{\Psi}_0(t)$ is the interacting vacuum state in the interaction representation, $\bar{\Psi}(t)$ is the real state-vector in the interaction representation, and $\psi_e(x)$ and $\phi_{\alpha}(y)$ are interaction representation operators for the nucleon and meson fields, respectively. x and y need not be on the space-like surface t . $\bar{\Psi}(t)$ and $\bar{\Psi}_0(t)$ satisfy the Schrodinger equation

$$i \frac{d\bar{\Psi}(t)}{dt} = H_I(t) \bar{\Psi}(t) \quad (2)$$

It is a consequence of equations 1 and 2 that

$$\begin{aligned} \phi_{\alpha}(x, y; t) - \phi_{\alpha}(x, y; -\infty) = & - \int_{-\infty}^t dt' \int_{-\infty}^{t'} dt'' \\ & (\bar{\Psi}_0^*(t'') [H_I(t''), [H_I(t'), \psi_e(x) \phi_{\alpha}(y)]] \bar{\Psi}(t'')) \end{aligned} \quad (3)$$

⁽⁴⁾ M. Cini. Nuovo Cimento 10 526 (1953)

if one requires that the initial state $\Psi(-\infty)$ be a state with one meson present.

We choose equation (3) as the fundamental equation for our system, because it will yield results in a simple way which are equivalent to the lowest-order TD approximation. The Tamm-Dancoff approximation as applied to equation (3) may be stated in the following manner. One arranges the operators occurring in the integrand in the normal order, (absorption operators to the right of emission operators) yielding a sum of terms of the general form of $\phi(x,y;t)$, possibly containing more than two field operators. We then drop all the terms except those having exactly the form of $\phi(x,y;t)$. That this procedure is equivalent to a Tamm-Dancoff approximation may be seen in the following way. A diagram is drawn in figure 1 which is a convenient way to enumerate the non-zero matrix elements of the interaction Hamiltonian H_1 . Since H_1 contains two nucleon field operators, and one meson operator, the non-zero elements are only those between states which differ by zero or two nucleons and by one meson. The lines in Fig. 1 represent matrix elements which connect the "ground state" (1 meson, 1 nucleon in state Ψ in excess of those in Ψ_0) to other states. It is clear that our approximation is equivalent to the Tamm-Dancoff approximation which restricts the allowed states of the system to the solid circles in Fig. 1.

When the above procedure has been carried out, we shall be left with an integral equation for $\phi_{\rho\alpha}(x,y;t)$ similar in appearance to the lowest order Bethe-Salpeter equation for the meson-nucleon system, with the important difference that our field operators are in the interaction representation, and their time-dependence is known.

Inserting $H_I(x_0) = \int d^3x H_I(x)$

$$H_I(x) = iG \bar{\psi}(x) \gamma_5 \tau_\alpha \phi_\alpha(x) \psi(x) \quad (4)$$

into Eq. (3), we obtain

$$\phi_{\alpha}(x, y; t) - \phi_{\alpha}(x, y, -\infty) = G \int_{-\infty}^t dz \int_{-\infty}^z dw \quad (5)$$

$$(\Psi_0^*(w_0) [\bar{\psi}(w) \gamma_5 \tau_\alpha \phi_\alpha(w) \psi(w), [\bar{\psi}(z) \gamma_5 \tau_\beta \phi_\beta(z) \psi(z), \psi_p(x) \phi_\alpha(y)]] \Psi(w_0))$$

$$(here \int_{-\infty}^t dz \text{ means } \int_{-\infty}^t dz_0 \int d^3z)$$

The double commutator in (5) may, without much difficulty, be put into normal order; if one retains only those terms which correspond to a one-meson, one-nucleon amplitude, it becomes

$$-3i S(x-z) \gamma_5 (1 + \varepsilon(z-w)P) [\langle \phi(w) \phi(z) \rangle_0 \langle \bar{\psi}(w) \psi(z) \rangle_0] \gamma_5 \phi_\alpha(y) \psi(w) \quad (6(a))$$

$$-2i \Delta(y-z) (1-P) S_p \left\{ \gamma_5 \langle \bar{\psi}(w) \psi(z) \rangle_0 \gamma_5 \langle \psi(w) \bar{\psi}(z) \rangle_0 \right\} \phi_\alpha(w') \psi_p(x) \quad (6(b))$$

$$\left\{ \langle \phi(z) \phi(y) \rangle_0 \langle \bar{\psi}(z) \psi(x) \rangle_0 + \langle \phi(y) \phi(z) \rangle_0 \langle \psi(x) \bar{\psi}(z) \rangle_0 \right\} \times \gamma_5 (1 + \varepsilon(z-w)P) \langle \bar{\psi}(w) \psi(z) \rangle_0 \gamma_5 \tau_\alpha \tau_\beta \phi_\beta(w) \psi(w) \quad (6(c))$$

$$i \Delta(y-z) \left\{ \langle \bar{\psi}(w) \psi(z) \rangle_0 \gamma_5 \langle \psi(w) \bar{\psi}(z) \rangle_0 - \langle \psi(x) \bar{\psi}(w) \rangle_0 \gamma_5 \langle \bar{\psi}(z) \psi(w) \rangle_0 \right\} \times \gamma_5 \tau_\beta \tau_\alpha \phi_\beta(w) \psi(z) \quad (6(d))$$

$$-i S(x-z) \left\{ \langle \phi(w) \phi(y) \rangle_0 \gamma_5 \langle \bar{\psi}(w) \psi(z) \rangle_0 + \langle \phi(y) \phi(w) \rangle_0 \gamma_5 \langle \psi(z) \bar{\psi}(w) \rangle_0 \right\} \times \gamma_5 \tau_\beta \tau_\alpha \phi_\beta(z) \psi(w) \quad (6(e))$$

By application of the usual rules for drawing the Feynman diagram corresponding to a given S-matrix element, the terms (6(a)) - (6(e)) may be seen to correspond to the time-ordered graphs drawn in Fig. 2.

It will be noticed that the graphs (a), (b), and (c) occur alone, without their negative-energy intermediate state analogs, and that the vacuum self-energy fails to appear at all, in contrast to what one finds with the old Tamm-Dancoff method. The reason that these terms do not appear is that the structure of the commutator in Eq. (5) is such that any Feynman diagram representing parts of it must have a line connecting z with x or y , or both.

If we next insert the propagation functions defined by ⁽⁵⁾

$$\begin{aligned}
 P\langle \phi(x) \phi(y) \rangle_0 &= \frac{1}{2} \Delta_F(x-y) \\
 \varepsilon(x-y) P\langle \psi(x) \bar{\psi}(y) \rangle_0 &= -\frac{1}{2} S_F(x-y) \\
 \langle \psi(z) \bar{\psi}(y) \rangle_0 &= -i S^T(z-y) \\
 &= -i S_A^T(z-y) \quad z_0 < y_0 \\
 &= +i S_{12}^T(z-y) \quad z_0 > y_0 \\
 \langle \bar{\psi}(y) \psi(z) \rangle_0 &= -i S_A^-(z-y) \quad z_0 < y_0 \\
 &= +i S_{12}^-(z-y) \quad z_0 > y_0 \\
 \langle \phi(z) \phi(y) \rangle_0 &= i \Delta_A^+(z-y) \quad z_0 < y_0 \\
 &= -i \Delta_R^+(z-y) \quad z_0 > y_0 \\
 \langle \phi(y) \phi(z) \rangle_0 &= -i \Delta_A^-(z-y) \quad z_0 < y_0 \\
 &= +i \Delta_R^-(z-y) \quad z_0 > y_0
 \end{aligned} \tag{7}$$

for the

vacuum expectation values in 6(d) and 6(e) which depend on field operators at w and z , we may extend the integration over w in (5) to all space-time, because 6(a), 6(b), and 6(c) are already zero when $z_0 < w_0$. The result

of this substitution is that (5) becomes

$$\begin{aligned}
 \phi_\alpha(x, y; t) - \phi_\alpha(x, y; -\infty) &= iG^2 \int_{-\infty}^t d\tau \int_{-\infty}^{\infty} d\omega \\
 &\left\{ \left[3 S(x-z) \gamma_5 \Delta(z-\omega) S(z-\omega) + \frac{3}{4} S(x-z) \gamma_5 \Delta(z-\omega) S_1(z-\omega) \gamma_5 \right] \phi_\alpha(\omega, y, \omega_0) \right. \\
 &+ \Delta(y-z) S_P \left\{ 2 \gamma_5 S(z-\omega) \gamma_5 S^*(\omega-z) \right. \\
 &\quad \left. + \frac{1}{2} \gamma_5 S_1(z-\omega) \gamma_5 S_F(\omega-z) \right\} \phi_\alpha(x, \omega; \omega_0) \\
 &+ \frac{i}{2} \left\{ -\Delta(y-z) S(x-z) + \Delta(y-z) S^*(x-z) \right\} \gamma_5 \\
 &\quad \times \left\{ 2i S(z-\omega) + S_F(z-\omega) \right\} \gamma_5 \tau_\alpha \tau_\beta \phi_\beta(\omega, \omega, \omega_0) \quad (8) \\
 &+ \Delta(y-z) \left\{ -S(x-\omega) \gamma_5 S_A^*(\omega-z) + S^*(x-\omega) \gamma_5 S_A^-(\omega-z) \right\} \\
 &\quad \times \gamma_5 \tau_\beta \tau_\alpha \phi_\beta(z, \omega, \omega_0) \\
 &+ S(x-z) \left\{ -\Delta(y-\omega) \gamma_5 S_2^-(z-\omega) + \Delta(y-\omega) \gamma_5 S_2^+(z-\omega) \right\} \\
 &\quad \times \gamma_5 \tau_\beta \tau_\alpha \phi_\beta(\omega, z, \omega_0) \left. \right\}
 \end{aligned}$$

III. Reduction to the non-covariant form.

It will be noticed from the form of $\phi(x, y, t)$ that it is non-zero in four different physical situations⁽³⁾. Specifically, decomposing ψ and ϕ into their positive and negative energy parts,

$$\phi_{\alpha\alpha}(x, y; t) = \phi^{++} + \phi^{+-} + \phi^{-+} + \phi_{\alpha\alpha}^{--}(x, y; t) \quad (9)$$

where, for example,

$$\phi_{\alpha}^{-+}(x, y; t) = (\bar{\Psi}_0^*(t) \phi_{\alpha}(x) \psi(y) \bar{\Psi}(t)).$$

This decomposition may be done explicitly here because the time dependence of the interaction representation operators ϕ and ψ is known. A non-zero ϕ^{++} corresponds to having a meson and a nucleon in the state $\bar{\Psi}$ in excess of those in $\bar{\Psi}_0$; ϕ^{-+} to one more nucleon and one less meson; ϕ^{+-} to one more meson and one less nucleon, ϕ^{--} to one less meson and one less nucleon in $\bar{\Psi}$ than $\bar{\Psi}_0$. We will now operate from the left on Eq. (9) with a positive energy projection operator for both nucleons and mesons, yielding a somewhat simplified equation:

$$\begin{aligned} \phi_{\alpha}^{+-}(x, y; t) - \phi_{\alpha}^{+-}(x, y; -\infty) &= iG^2 \int_{-\infty}^t dz \int d\xi \\ &\left\{ \left[3 S^+(x-z) \gamma_5 \Delta^-(\xi) S(\xi) \gamma_5 + \frac{3}{4} S^+(x-z) \Delta^-(\xi) \gamma_5 S_1(\xi) \gamma_5 \right] \right. \\ &\times [\phi_{\alpha}^{++} + \phi_{\alpha}^{--}](z-\xi, y; z_0-\xi_0) + \Delta^+(y-z) S_P \left\{ 2 \gamma_5 S(\xi) \gamma_5 S(-\xi) \right. \\ &\quad \left. - \frac{1}{2} \gamma_5 S_1(\xi) \gamma_5 S_1(-\xi) \right\} [\phi_{\alpha}^{++} + \phi_{\alpha}^{--}](x, z-\xi, z_0-\xi_0) \\ &\quad + \frac{1}{2} \Delta^+(y-z) S^+(x-z) \gamma_5 (2 S(\xi) + S_1(\xi)) \gamma_5 \tau_x \tau_0 \phi_{\beta}(z-\xi, z-\xi, z_0-\xi_0) \\ &\quad + \Delta^+(y-z) S^+(x-z+\xi) \gamma_5 S_A(-\xi) \gamma_5 \tau_0 \tau_x \phi_{\beta}(z, z-\xi, z_0-\xi_0) \\ &\quad \left. + S^+(x-z) \Delta^+(y-z+\xi) \gamma_5 S_2^+(\xi) \gamma_5 \tau_0 \tau_x \phi_{\beta}(z-\xi, z, z_0-\xi_0) \right\} \end{aligned} \quad (10)$$

This is exactly the equation which would have been obtained if we had started with ϕ^{++} in equation 3. We will next drop all terms on the right-hand side of eq. (10) which contain either meson or nucleon negative-energy amplitudes. This is consistent with the Tamm-Dancoff approximation; the dropped components of the wave-function are not among those connected on figure 1 to the ground state.

The terms in eq. (10) containing $\Delta(\xi) S(\xi)$, $S(\xi) S(-\xi)$ and $S(\xi)$ will drop out in the integration over ξ_0 .

After dropping the negative-energy amplitudes, we take the Fourier transform of equation (10). The expansion of ϕ^{++} is found to be

$$\phi^{++}(x, y; t) = \int \frac{d^3\xi d^3r}{(\omega_\xi E_r)^{1/2}} e^{i r \cdot x + i \xi \cdot y} \times \sum_{u, \alpha} u a(ru, q\alpha; t) \quad (11)$$

where $r \cdot x = \vec{r} \cdot \vec{x} - r_0 x_0$, $E_r = (M^2 + r^2)^{1/2}$, $\omega_\xi = (\mu^2 + \xi^2)^{1/2}$

(M and μ are the masses of the nucleon and meson), and

$$a(ru, q\alpha; t) = (\bar{\Psi}(t) b_{ru} a_{q\alpha} \Psi(t)) \quad \text{Here}$$

b_{ru} and $a_{q\alpha}$ are annihilation operators for the nucleon and meson, respectively. Inserting the time dependence of the state vectors,

$$\begin{aligned} \bar{\Psi}(t) &= e^{-i(\bar{E} - H_0)t} \bar{\Psi}(0) \\ \bar{\Psi}_0(t) &= e^{-i(\bar{E}_0 - H_0)t} \bar{\Psi}_0(0) \end{aligned} \quad (13)$$

where H_0 is the no-interaction Hamiltonian, \bar{E} is the total energy of the system, and \bar{E}_0 is the vacuum state energy, equation 12 becomes ($\varepsilon = \bar{E} - \bar{E}_0$)

$$a(ru, q\alpha, t) = e^{-i(\varepsilon - E_r - \omega_\xi)t} a(ru, q\alpha) \quad (14)$$

where $a(ru, q\alpha) = (\bar{\Psi}_0^*(0) b_{ru} a_{q\alpha} \bar{\Psi}_0(0))$.

With this notation, equation (10), written in momentum space in the center-of-mass system becomes (p = momentum of nucleon; $\Delta = \varepsilon - E_p - \omega_p$)

$$\begin{aligned}
 \Delta O(pu, -p\alpha) = & -\frac{3}{4} i G^2 \frac{M}{E_F} \int d^4 \xi (\bar{\chi}(\xi) \gamma_5 S_F(\xi) \chi(-\xi)) \Delta_F(\xi) \\
 & \times e^{-i \vec{p} \cdot \vec{\xi} + i(\Delta + E_F) \xi_0} \alpha(pu, -p\alpha) \\
 & - \frac{1}{4} i G^2 \frac{1}{\omega_p} \int d^4 \xi S_p \left\{ \gamma_5 S_F(\xi) \gamma_5 S_F(-\xi) \right\} e^{-i \vec{p} \cdot \vec{\xi} + i(\Delta + \omega_p) \xi_0} \alpha(pu, -p\alpha) \\
 & + \frac{M^2 G^2}{16 \pi^3} \int \frac{d^3 k}{(E_p \omega_p E_k \omega_k)^{1/2}} \left\{ \left[\frac{(\bar{u}(p) \gamma_5 \Lambda_+(0) \gamma_5 u(k))}{M(M-\xi)} \right. \right. \\
 & + \left. \left. \frac{(\bar{u}(p) \gamma_5 \Lambda_-(0) \gamma_5 u(k))}{M(M+\xi)} \right] \tau_\alpha \tau_\beta + \left[\frac{(\bar{u}(p) \gamma_5 \Lambda_+(-i-k) \gamma_5 u(k))}{E_{p+k} (E_p + E_{p+k} + E_k - \xi)} \right. \right. \\
 & + \left. \left. \frac{(\bar{u}(p) \gamma_5 \Lambda_+(-i-k) \gamma_5 u(k))}{E_{p+k} (E_{p+k} + \omega_k + \omega_p - \xi)} \right] \tau_\alpha \tau_\beta \right\} \alpha(ku, -k\beta)
 \end{aligned} \tag{15}$$

Here we have, for convenience in the next section, left the self-energy parts in terms of invariant functions. Except for these terms, and for the terms involving the energy denominator $1/M \pm \xi$, which contributes only to the states for which $I = \bar{I} = 1/2$, this equation is identical to that derived by the old Tamm-Dancoff method ⁽¹⁾. We may therefore evaluate these self-energy parts, and use them to find corrections to the phase shifts previously calculated, except for the $I = \bar{I} = 1/2$ states.

IV. Renormalization.

$$\text{Let } \Sigma(\xi) = \gamma_5 S_F(\xi) \gamma_5 \Delta_F(\xi)$$

(16)

$$\text{and } \Sigma(\xi) = S_{p \text{ or } r} \left\{ \gamma_5 S_F(\xi) \gamma_5 S_F(-\xi) \right\}$$

These are the well-known nucleon and meson self-energy propagation factors from second-order perturbation theory whose strong singularity at $\xi = 0$ causes the divergence of their momentum-space integrals. We shall follow the usual relativistic subtraction procedures to extract their finite parts,

We take the 4-dimensional Fourier transform of Ω and Σ :

$$\begin{aligned}\Omega(p) &= \int e^{-ip \cdot \xi} \Omega(\xi) d^4 \xi \\ \Sigma(k) &= \int e^{-ik \cdot \xi} \Sigma(\xi) d^4 \xi\end{aligned}\tag{17}$$

These are now relativistically invariant functions of the 4-vector momenta p and k . More specifically, $\Omega(p)$ is a function of $p_{\mu} \gamma_{\mu}$ only, since $p_{\mu} \gamma_{\mu}$ is the simplest invariant matrix function of p . Similarly $\Sigma(k)$ is a function of k^2 only. These facts may be easily verified by explicit calculation.

In the usual way, (6) we set the observable (renormalized) parts of the self-energy operators equal to the following:

$$\Omega_R(p \cdot \gamma) = \Omega(p \cdot \gamma) - \Omega(iM) - (p \cdot \gamma - iM) \left[\frac{\partial \Omega}{\partial (p \cdot \gamma)} \right]_{p \cdot \gamma = iM} \tag{18}$$

$$\Sigma_R(k^2) = \Sigma(k^2) - \Sigma(-\mu^2) - (k^2 + \mu^2) \left[\frac{\partial \Sigma}{\partial k^2} \right]_{k^2 = -\mu^2} \tag{19}$$

(6) P.T. Matthews Phil. Mag. 41 185 (1950)

NUCLEON SELF-ENERGY RENORMALIZATION

The subtractions indicated in (19) may be carried out by following exactly the procedure of Karplus and Kroll⁽⁷⁾ for the electron self-energy.

$$\begin{aligned}\Omega(p, \gamma) &= \int e^{-i p \cdot \xi} \gamma_5 S_F(\xi) \gamma_5 \Delta_F(\xi) d^4 \xi \\ &= -\frac{4i}{(2\pi)^4} \int d^4 s \gamma_5 \frac{S \cdot \gamma + p \cdot \gamma + iM}{(p+s)^2 + M^2} \gamma_5 \frac{1}{s^2 + \mu^2}\end{aligned}\quad (20)$$

We use

$$\frac{1}{a+b} = \int_0^1 \frac{du}{[au + b(1-u)]^2} \quad (21)$$

to get

$$\Omega(p, \gamma) = \frac{4i}{(2\pi)^4} \int d^4 s \int_0^1 du \frac{S \cdot \gamma + p \cdot \gamma - iM}{\{[(p+s)^2 + M^2]u + [s^2 + \mu^2](1-u)\}^2} \quad (22)$$

The denominator of the integrand may be written $[(s+pu)^2 + \Lambda^2]^2$ where $\Lambda^2 = p^2(u-u^2) + M^2u + \mu^2(1-u)$.

We expand the integrand in a power series in pu ; keeping only the first two terms, because the higher ones may be transformed into vanishing surface integrals. This casts (22) into the form

$$\begin{aligned}\Omega(p, \gamma) &= \frac{4i}{(2\pi)^4} \int d^4 s \int_0^1 du \left[\frac{S \cdot \gamma + p \cdot \gamma(1-u) - iM}{[s^2 + \Lambda^2]^2} \right. \\ &\quad \left. + pu \cdot \frac{\partial}{\partial s} \frac{S \cdot \gamma}{[s^2 + \Lambda^2]^2} \right]\end{aligned}\quad (23)$$

(7) R. Karplus and N.M. Kroll Phys Rev 77 536 (1950)

Upon carrying out the differentiation in (23), and making use of the facts that

$$\int f(t^2) t \cdot x d^4 t = 0$$

$$\int f(t^2) (a \cdot t) (b \cdot t) d^4 t = \frac{1}{4} (a \cdot b) \int f(t^2) t^2 d^4 t \quad (24)$$

we find the following

$$\Omega(p \cdot x) = \frac{4}{(2\pi)^4} \int d^4 t \int_0^1 du \left[\frac{p \cdot x - iM}{(t^2 + \Lambda^2)^2} - \frac{ut^2}{(t^2 + \Lambda^2)^3} \right] \quad (25)$$

The subtraction procedures (18) may be applied to this equation, and, making use of (8)

$$\int d^4 t \left\{ \frac{1}{(t^2 + \Lambda^2)^2} - \frac{1}{(t^2 + \Lambda_0^2)^2} \right\} = -i\pi^2 \log \left| \frac{\Lambda^2}{\Lambda_0^2} \right|$$

$$= -i\pi^2 \int_0^1 d\omega \frac{\Lambda^2 - \Lambda_0^2}{\Lambda_0^2 + (\Lambda^2 - \Lambda_0^2)\omega} \quad (26)$$

one obtains

$$\Omega_2(p \cdot x) = \frac{1}{(2\pi)^2} (p \cdot x - iM)^2 \int_0^1 du \int_0^1 d\omega$$

$$\times \frac{u - u^2}{\Lambda_0^2 + (\Lambda^2 + M^2)(u - u^2)\omega} \times \left\{ (p \cdot x + iM) \left(1 - u - \frac{2M^2 u^2 (1-u)\omega}{\Lambda_0^2} \right) - iMu \right\} \quad (27)$$

where $\Lambda_0^2 = M^2 u^2 + \mu^2(1-u)$

It is worthwhile noting that, although it would greatly simplify the approximate evaluation of (27) to put $\mu^2 = 0$, it may not be done, because Σ_R is not an analytic function of μ^2 at $\mu^2 = 0$.

MESON SELF-ENERGY RENORMALIZATION

$$\begin{aligned} \Sigma(\kappa) &= \int e^{-i\kappa \cdot x} S_P \left\{ \gamma_5 S_F(\frac{x}{2}) \gamma_5 S_F(-\frac{x}{2}) \right\} d^4x \\ &= \frac{1}{4\pi^4} \int d^4p S_P \left\{ \gamma_5 \frac{p \cdot \gamma + iM}{p^2 + M^2} \gamma_5 \frac{(p-\kappa) \cdot \gamma + iM}{(p-\kappa)^2 + M^2} \right\} \end{aligned} \quad (28)$$

By evaluating the spur here, one may write

$$\begin{aligned} \Sigma(\kappa) &= -\frac{1}{8\pi^4} \int \frac{d^4p}{(p-\kappa)^2 + M^2} - \frac{1}{8\pi^4} \int \frac{d^4p}{p^2 + M^2} \\ &+ \frac{\kappa^2}{8\pi^4} \int \frac{d^4p}{[(p-\kappa)^2 + M^2][p^2 + M^2]} \end{aligned} \quad (29)$$

The second term clearly gives no contribution to Σ_R , since it will drop out in the first subtraction in (19). This is also true of the first term, as may be seen by expanding the integrand, as was done in (23).

We find

$$\int \frac{d^4p}{(p-\kappa)^2 + M^2} = \int \frac{d^4p}{p^2 + M^2} - \kappa^2 M^2 \int \frac{d^4p}{(p^2 + M^2)^3}$$

This will completely drop out in the subtraction (19). Therefore the observable parts of \sum are all contained in the third term of (29), \sum_3 .

Application of (21) yields

$$\sum_3 = \frac{\kappa^2}{8\pi^4} \int_0^1 du \int d^4s \frac{1}{[s^2 + u(1-u)\kappa^2 + M^2]^2} \quad (30)$$

after a change of variable $S = p - u\kappa$. We define

$$J(\Lambda) = \frac{\kappa^2}{8\pi^4} \int_0^1 du \int d^4s \left\{ \frac{1}{[s^2 + u(1-u)\kappa^2 + M^2]^2} - \frac{1}{[s^2 + u(1-u)\kappa^2 + \Lambda^2]^2} \right\} \quad (31)$$

where Λ is a very large mass.

Using (26), (31) becomes

$$J(\Lambda) = -\frac{1}{8\pi^2} \kappa^2 (M^2 - \Lambda^2) \int_0^1 du \int_0^1 dw \frac{1}{\Lambda^2 + \kappa^2(1-u)u + (M^2 - \Lambda^2)w} \quad (32)$$

The subtractions (19) may be conveniently performed on this expression; when one does them, and lets $\Lambda^2 \rightarrow \infty$, one obtains

$$\begin{aligned} \sum_R(\kappa^2) &= \frac{1}{8\pi^2} (\kappa^2 + \mu^2)^2 \int_0^1 du \int_0^1 dw \frac{u(1-u)}{M^2 - u(1-u)\mu^2} \\ &\times \left\{ \frac{\kappa^2 u(1-u)w}{u(1-u)w(\kappa^2 + \mu^2) + M^2 - u(1-u)\mu^2} - 1 \right\}, \quad (33) \end{aligned}$$

which may be quite accurately approximated by letting $\mu^2 = 0$ under the integral sign. One then obtains

$$\sum_R = -\frac{1}{8\pi^2} (\kappa^2 + \mu^2)^2 \int_0^1 du \frac{1}{\kappa^2} \log \left| 1 + u(1-u) \frac{\kappa^2}{M^2} \right| \quad (34)$$

V. Conclusion

The integrals in (27), (34) are elementary, and may be carried out without difficulty. The arguments of Ω and Σ are replaced by the following functions of momentum and energy in the center-of-mass system

$$p \text{ and } \varepsilon; \quad p_\mu = (\vec{p}, \Delta + E_p), \text{ and} \\ \kappa_\mu = (-\vec{p}, \Delta + \omega_p)$$

Thus $(p \cdot \gamma - iM)^2$ becomes $-\Delta^2$ and $(p \cdot \gamma - iM)^2 (p \cdot \gamma + iM)$ becomes $-i\Delta \frac{E_p}{M} (\Delta + 2E_p)$; since what occurs in (15) is $(\bar{u}(p) \Omega(\xi) u(p))$ and $u(p)$ satisfies

$$(\vec{p} \cdot \vec{\gamma} + i\gamma_4 E_p - iM) u(p) = 0 \\ (\bar{u}(p) u(p)) = 1 \quad (35)$$

Eq. (15) may now be rewritten in the form

$$\Delta u(p_1, p_2) = \left(\frac{G^2}{4\pi} \right)_{\text{effective}} \times (\text{Interaction terms}) \quad (36)$$

where the interaction terms include all those on the right-hand-side of (15) except the self-energy terms, and

$$\left(\frac{G^2}{4\pi} \right)_{\text{effective}} = \frac{G^2}{4\pi} \left\{ 1 + \frac{3iG^2 M}{4 E_p} \Omega + \frac{iG^2}{4 \omega_p} \Sigma \right\}^{-1} \quad (37)$$

The effect of the self-energy terms may therefore be considered as causing the coupling constant to become a function of momentum and energy.

When one calculates the asymptotic expressions for the integrals (34) and (27) for large p and inserts them into (37), the latter becomes

$$\left(\frac{G^2}{4\pi} \right)_{\text{effective}} = \frac{G^2}{4\pi} \left\{ 1 - \frac{3G^2}{32\pi^2} \log \frac{p}{M} - \frac{G^2}{32\pi^2} \frac{\varepsilon}{p} \log \frac{p}{M} \right\} \quad (38)$$

Here the first term within the brackets arises from the nucleon self-energy, the second from the meson. It is seen that for large p (38) is negative.⁽⁹⁾ For $\Delta=0$ it is by definition positive and equal to $G^2/4\pi$. Therefore for some $p = K$ for which $\Delta < 0 \left(\frac{G^2}{4\pi} \right)_{eff}$ has a pole. A pole in the wave function $\alpha(p_u, -p_u)$ would imply that an outgoing wave of momentum K was present, if one assumed an ingoing wave for which $\Delta = 0$. This K , however, with the given energy of the system Σ , would form a space-like energy-momentum four-vector, which is highly unphysical, corresponding to something like production of particles of imaginary mass. No reconciliation of this nonsensical prediction with reality has yet been made, and at present it must be considered as raising serious doubt about the validity or consistency of the Tamm-Dancoff approximation.

I would like to thank F. J. Dyson for suggesting this problem, for essential guidance, and for several helpful discussions.

(9) R. H. Dalitz (Private communication) has pointed out an error made by me in the evaluation of these terms which radically alters their behavior. His information has prevented the possible publication of qualitatively wrong results, and is greatly appreciated.

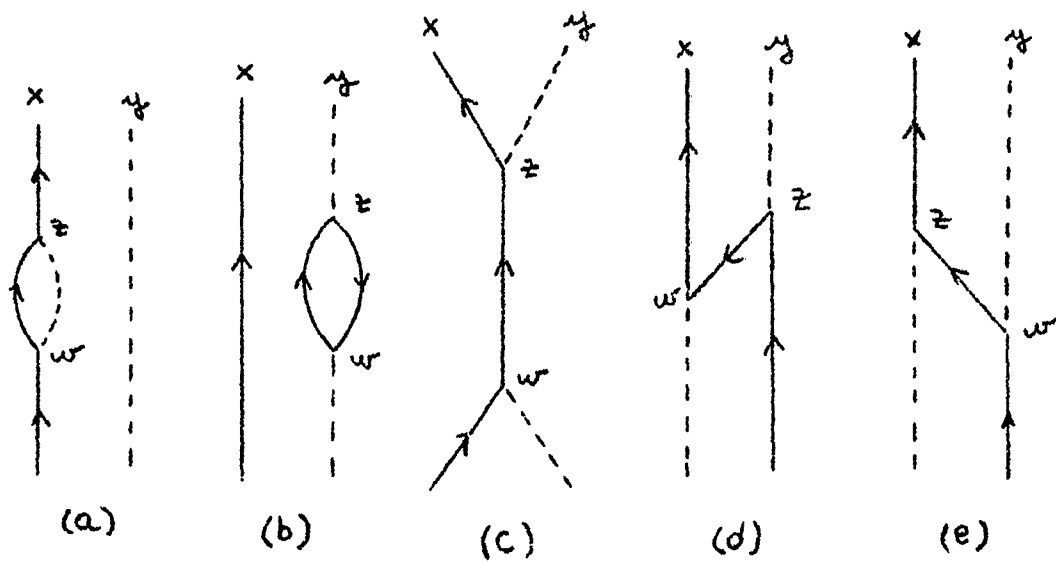


Figure 2. Diagrams corresponding to the terms of equation 6. Time increases vertically.

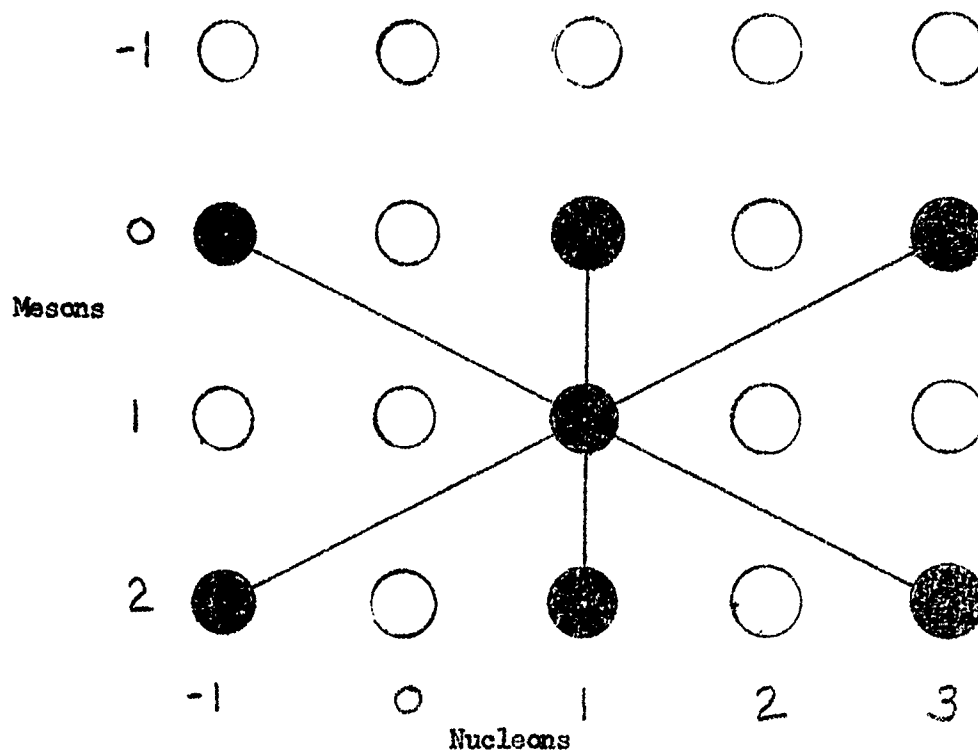


Figure 1. Intermediate states of the meson - nucleon scattering system which are allowed in the Tamm-Dancoff approximation. Vertically is plotted the number of mesons present in the real state in excess of those present in the vacuum, horizontally the number of nucleons.